

# POVRŠINSKI NAPON

# Priroda

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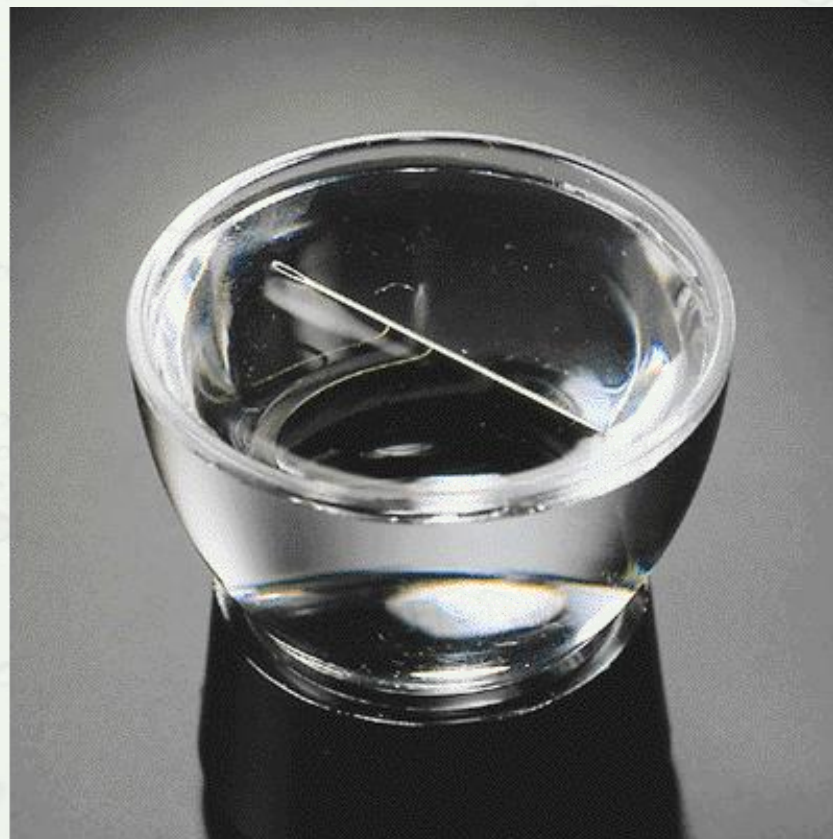
Zašto ovaj insekt može da hoda po površini vode?

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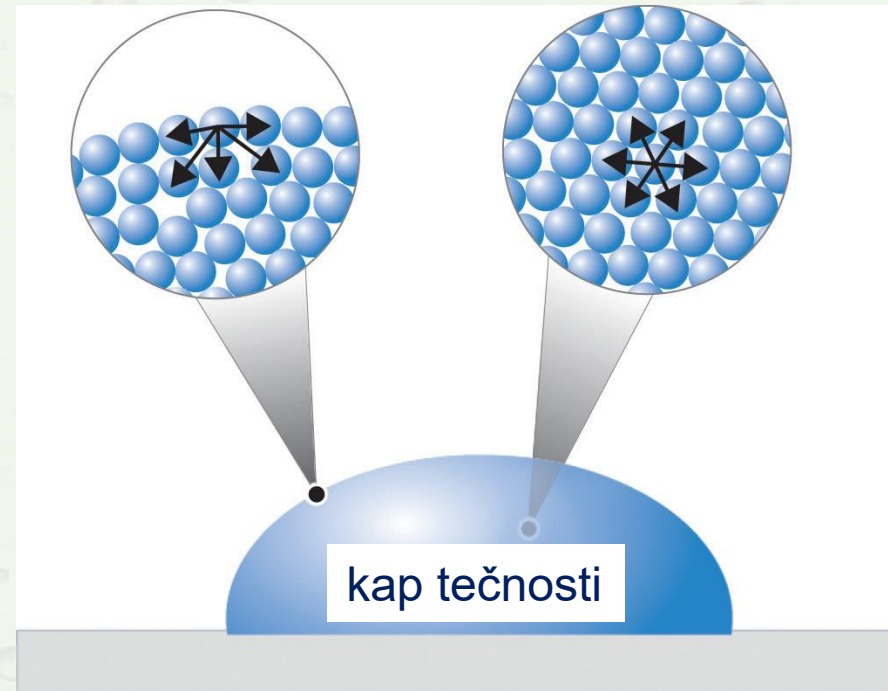
# Zašto neki mali predmeti plivaju po površini vode?

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# Površinski napon

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jake međumolekulske interakcije



veliki površinski napon

Potrebno je uložiti energiju da se molekul iz unutrašnjosti dovede na površinu.

# Definicija

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$$p = \text{const}, T = \text{const}, n = \text{const} : dG = \gamma d\mathcal{A}$$

$$V = \text{const}, T = \text{const}, n = \text{const} : dA = \gamma d\mathcal{A}$$

$\gamma$  - površinski napon

$$\gamma = \left( \frac{\partial G}{\partial \mathcal{A}} \right)_{p,T,n}$$

$$\gamma = \left( \frac{\partial A}{\partial \mathcal{A}} \right)_{V,T,n}$$

# Površinska Gibsova slobodna energija i površinska entalpija

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$$\gamma = \left( \frac{\partial G}{\partial \mathcal{A}} \right)_{P,T,n} = G^s$$

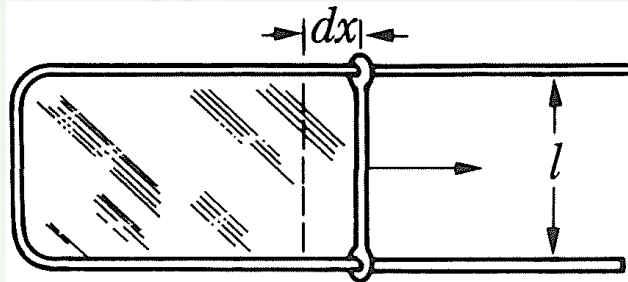
površinska Gibsova  
slobodna energija

$$H^s = \gamma - T \frac{d\gamma}{dT}$$

površinska entalpija

# Obrazovanje površine: rad i sila

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$$d\mathcal{A} = 2 \cdot l dx$$

$$dW = \gamma d\mathcal{A}$$

$\gamma$  – površinska energija (J/m<sup>2</sup>)

$$|\vec{F}| = \frac{dW}{dx} = \frac{\gamma d\mathcal{A}}{dx} = 2\gamma l$$

$\gamma$  – površinski napon (N/m)



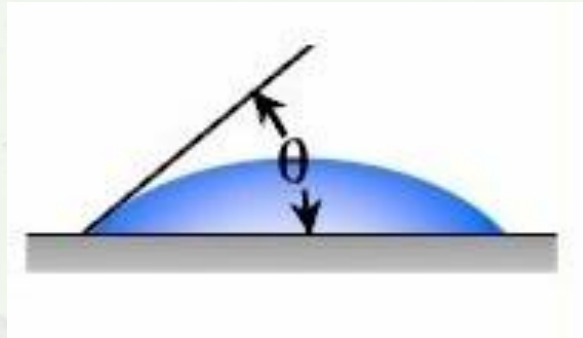
# Površinski napon odabranih tečnosti

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tečnost	površinski napon (mN/m)	temperatura (°C)
neon	5,2	-247
kiseonik	15,7	-193
etanol	22,3	20
maslinovo ulje	32,0	20
voda	72,8	20
živa	465,0	20
srebro	800,0	9070
zlato	1000,0	1070
bakar	1100,0	1130

# Ugao dodira

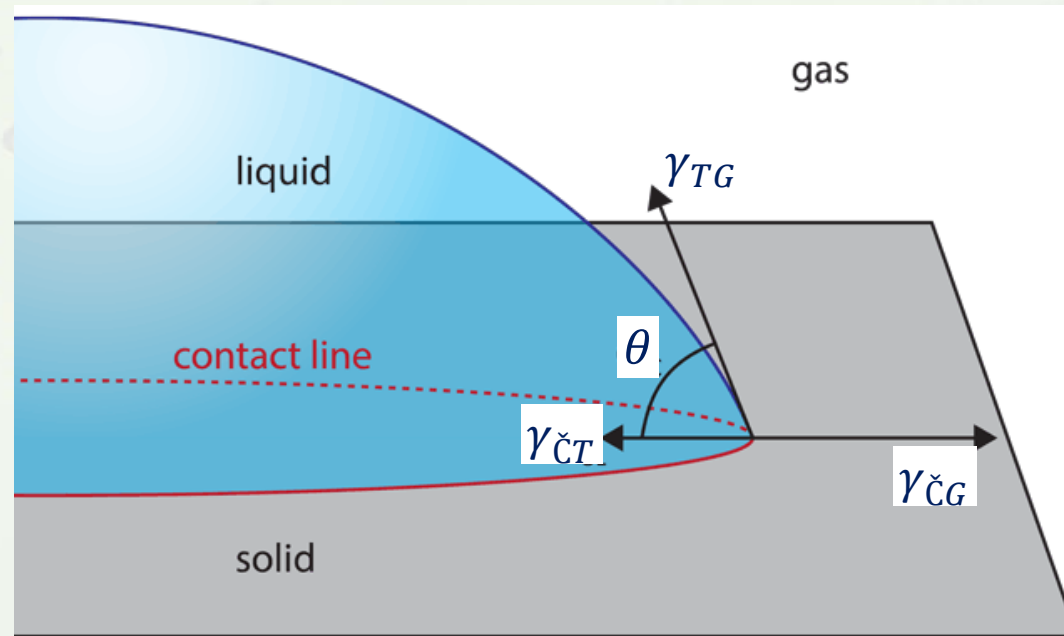
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**Ugao dodira** – ugao između meniska tečnosti i zida suda u kome se tečnost nalazi (meren u tečnosti).

# Ugao dodira

Ugao dodira se definiše iz ravnoteže sila na graničnu liniju između G, T i Č faza u horizontalnoj ravni:



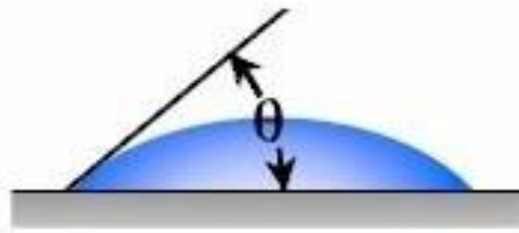
Jangova jednačina:

$$\gamma_{\check{C}G} = \gamma_{\check{C}T} + \gamma_{TG} \cos \theta$$

# Ugao dodira

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$$\gamma_{\check{c}G} = \gamma_{\check{c}T} + \gamma_{TG} \cos \theta$$

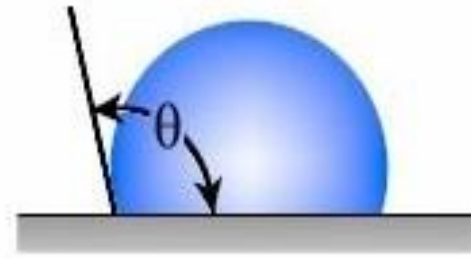


tečnost kvasi površinu

$$\gamma_{\check{c}G} > \gamma_{\check{c}T}$$

$$\cos \theta > 0$$

$$\theta < 90^{\circ}$$



tečnost ne kvasi površinu

$$\gamma_{\check{c}G} < \gamma_{\check{c}T}$$

$$\cos \theta < 0$$

$$90^{\circ} < \theta < 180^{\circ}$$

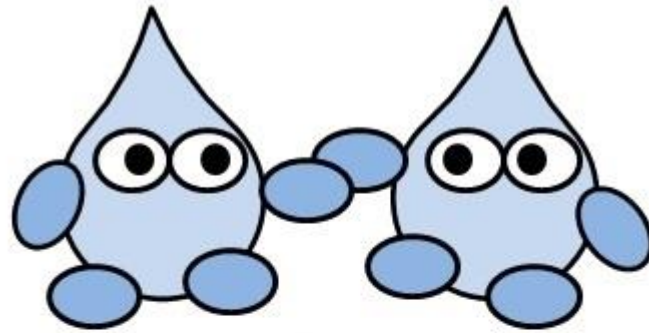
# Ugao dodira i kvašenje površine

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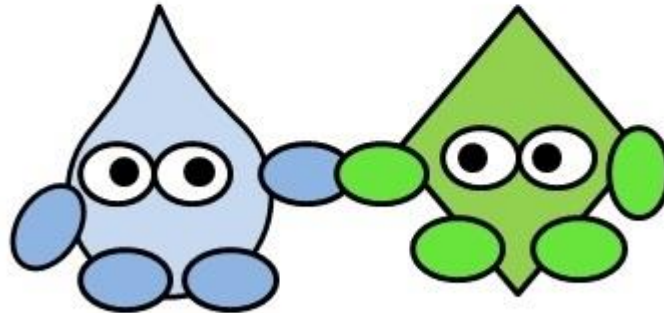


# Kohezija i adhezija sile

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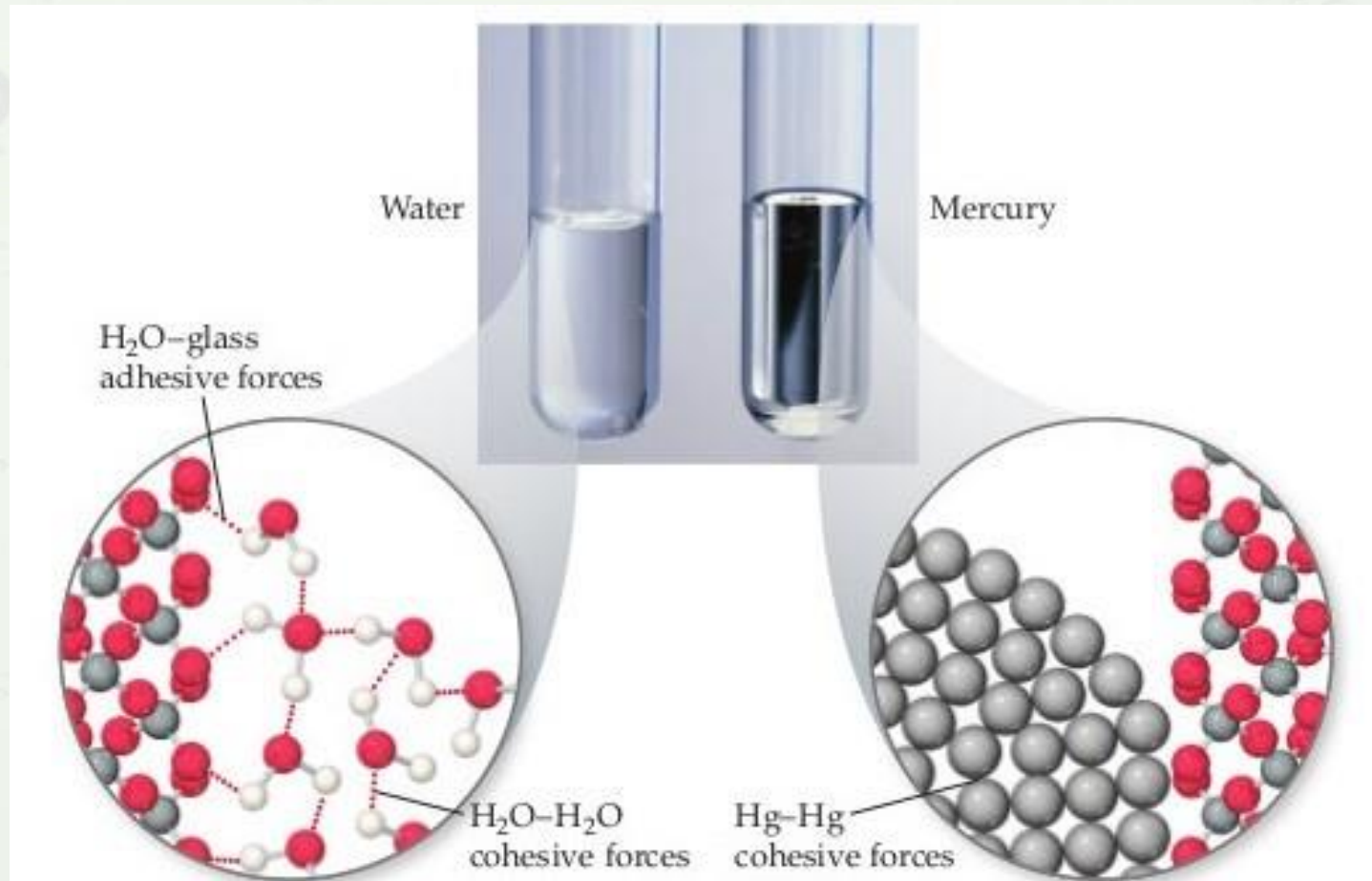
kohezija



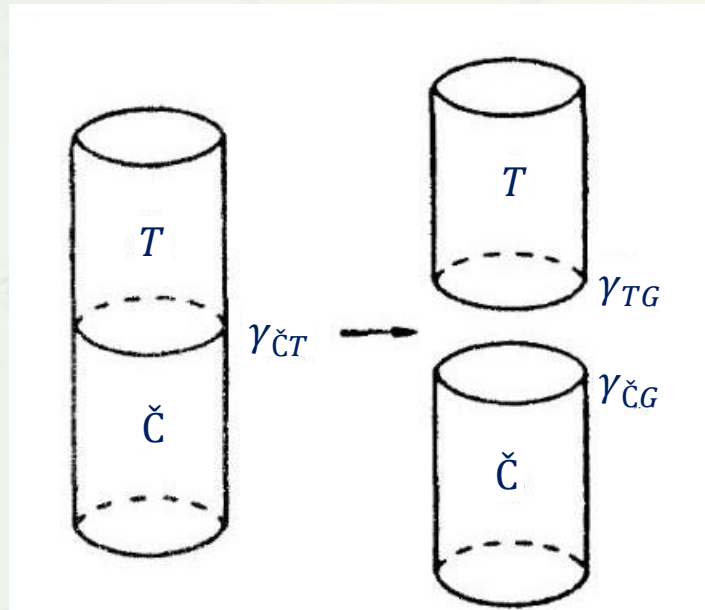
adhezija

# Voda – staklo i živa – staklo

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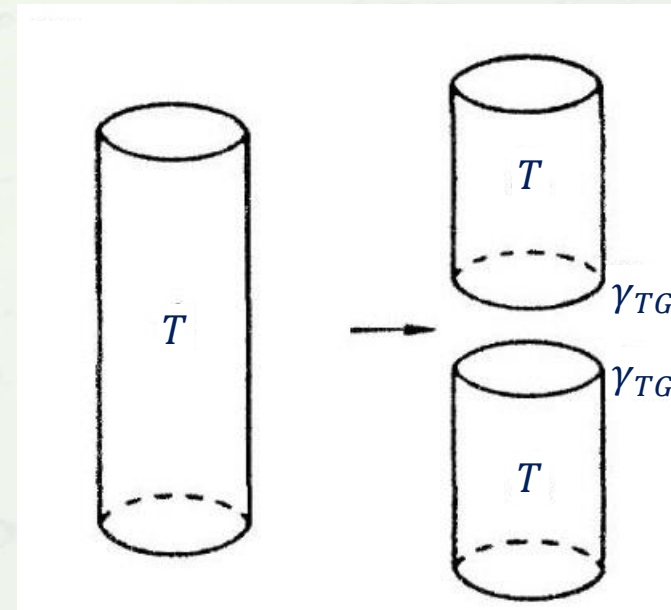


# Athezioni i kohezioni rad



**Athezioni rad** (Dipreova jednačina)

$$w_{\check{c}T} = \gamma_{\check{c}G} + \gamma_{TG} - \gamma_{\check{c}T}$$



**Kohezioni rad**

$$w_{TT} = 2\gamma_{TG}$$



# Ugao dodira

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$$\left. \begin{aligned} \gamma_{\check{C}G} &= \gamma_{\check{C}T} + \gamma_{TG} \cos \theta \\ w_{\check{C}T} &= \gamma_{\check{C}G} + \gamma_{TG} - \gamma_{\check{C}T} \\ w_{TT} &= 2\gamma_{TG} \end{aligned} \right\} \cos \theta = \frac{w_{\check{C}T}}{w_{TT}/2} - 1$$

tečnost kvasi površinu

$$\theta < 90^{\circ}, w_{\check{C}T} > w_{TG}/2$$

tečnost ne kvasi površinu

$$90^{\circ} < \theta < 180^{\circ}, w_{\check{C}T} < w_{TG}/2$$

# Živa na staklu

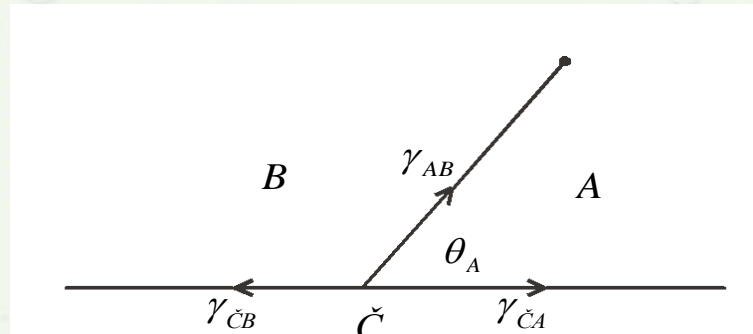
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$$\theta=140^{\circ} \rightarrow w_{\check{C}T} / (w_{TT} / 2) = 0,23$$

# Nemešljive tečnosti: razastiranje tečnosti

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$$\gamma_{\check{C}B} = \gamma_{\check{C}A} + \gamma_{AB} \cos \theta_A$$



# Razastiranje tečnosti

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Dve nemešljive tečnosti A i B, tečnost A razastire se spontano po tečnosti B:

$$\gamma_{AB} + \gamma_A - \gamma_B < 0$$

Athezioni rad između A i B:  $w_{AB} = \gamma_A + \gamma_B - \gamma_{AB}$

Uslov za razastiranje:  $w_{AB} > 2\gamma_A$

Koeficijent razastiranja:  $\gamma_B - \gamma_A - \gamma_{AB}$

# Za datu zapreminu tečnosti površina se može smanjiti formiranjem **zakrivljene površine**

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**Kapljica:** mala zapremina tečnosti  
u ravnoteži sa okružujućom parom.



**Mehur:** šupljina u tečnosti ispunjena  
parom.



**Balon:** oblast u kojoj je para zarobljena  
tankim filmom koji ima dve površine.

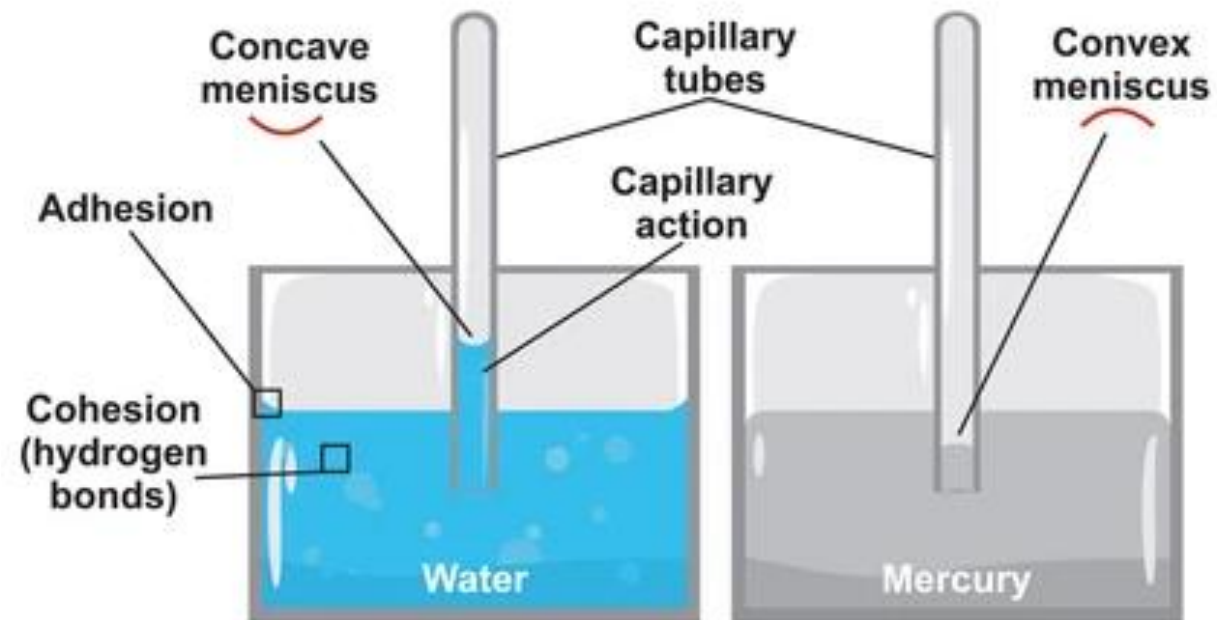
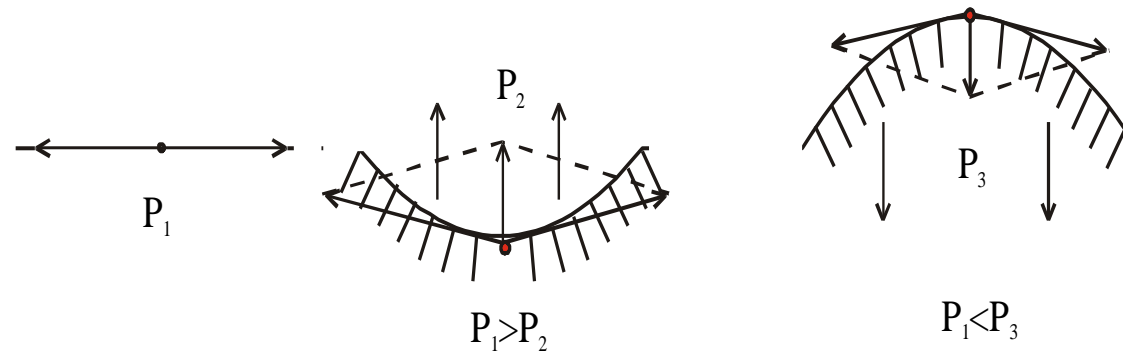


# Zakrivljena površina – posledice

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- kapilarne pojave
- napon pare tečnosti zavisi od zakrivljenosti površine

# Kapilarne pojave



# Kapilarne pojave

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$$dG_1 = -\gamma d(4\pi r^2) = -8\gamma\pi r dr$$

$$dG_2 = \Delta P d\left(\frac{4}{3}\pi r^3\right) = 4\Delta P\pi r^2 dr$$

$$dG = dG_1 + dG_2 = 0$$

$$-8\gamma\pi r dr + 4\Delta P\pi r^2 dr = 0$$

$$\Delta P = \frac{2\gamma}{r}$$

$$\Delta P = \gamma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

Laplasova jednačina



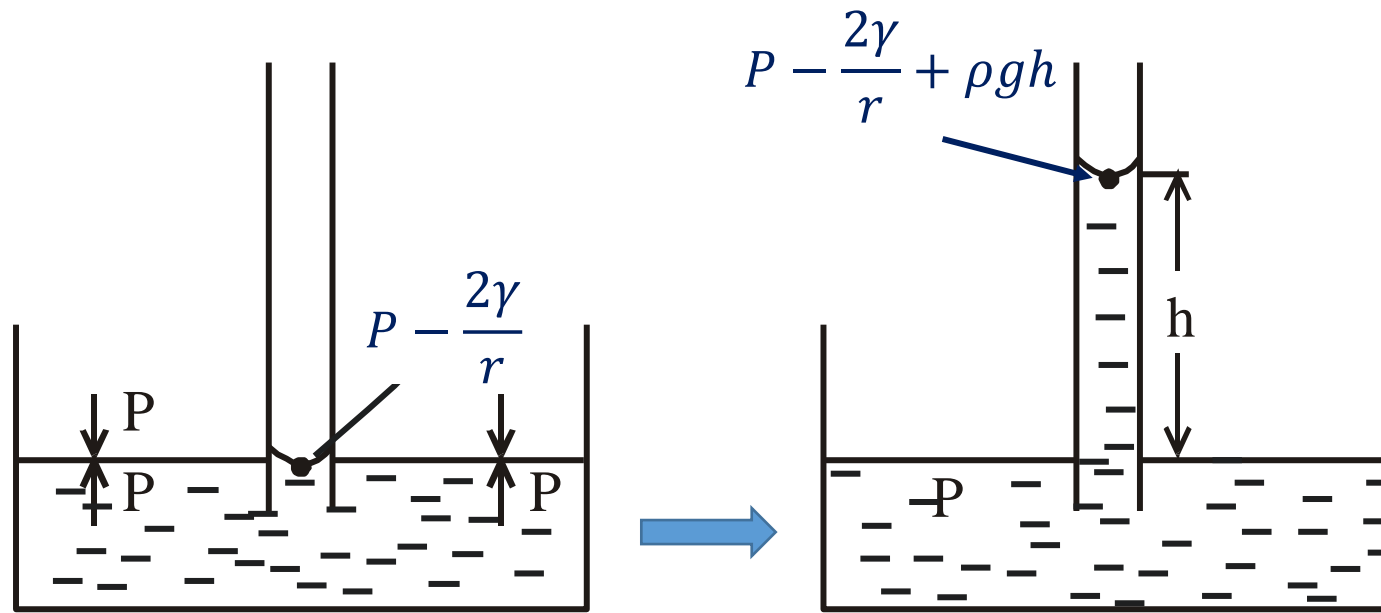
# Primer – mehur u vodi

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$$T = 298 \text{ K}$$
$$\gamma = 72 \text{ mN/m}$$

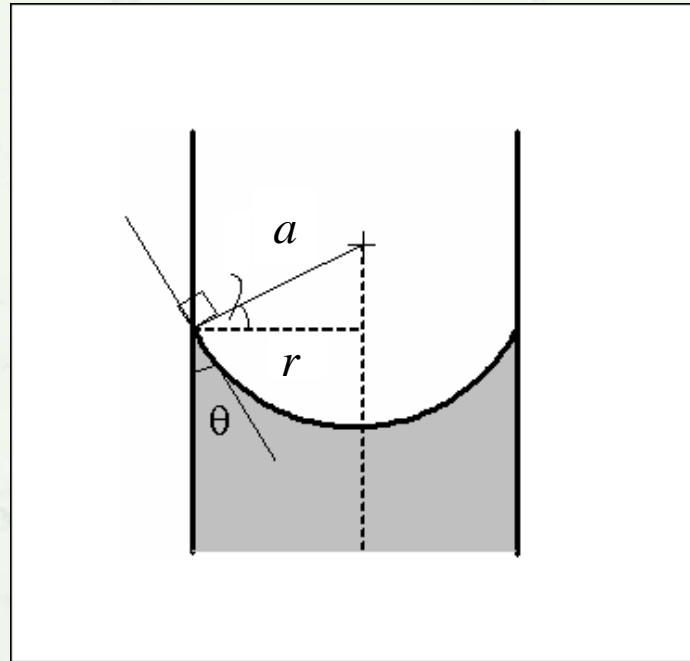
$2r / \mu\text{m}$	$\Delta P / \text{Pa (atm)}$
1 000	288 (0,00284)
3,0	96 000 (0,947)
0,3	960 000 (9,474)

# Kapilarnost je posledica površinskog napona



# Kapilarnost

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$$\rho g h = \frac{2\gamma}{a} \quad \longrightarrow \quad \gamma = \frac{1}{2} \rho g h a$$

$$\gamma = \frac{1}{2} \rho g h \frac{r}{\cos \theta}$$

# Zakrivljena površina – posledice

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- kapilarne pojave
- napon pare tečnosti zavisi od zakrivljenosti površine

# Površinski napon i napon pare

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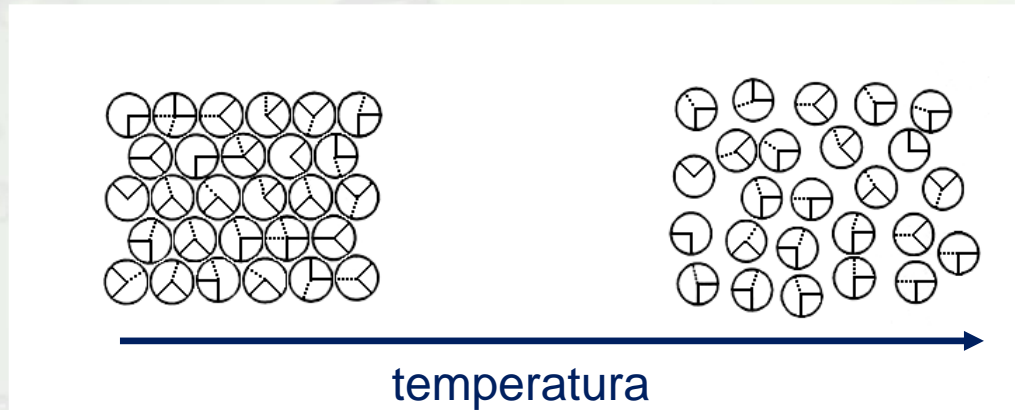
Promena molarne Gibsove slobodne energije pri obrazovanju zakrivljene površine:

$$\left. \begin{aligned} \Delta G_m(t) &= \int_0^{2\gamma/r} V_m^t dP = \frac{2\gamma V_m^t}{r} \\ \Delta G_m(g) &= RT \ln \frac{p}{p^0} \\ \Delta G_m(t) &= \Delta G_m(g) \end{aligned} \right\} \ln \frac{p}{p^0} = \frac{2\gamma M}{RT\rho r} = \frac{2\gamma V_m^t}{RT r}$$

$$p = p^0 \exp\left(\frac{2\gamma V_m^t}{RT r}\right)$$

Kelvinova jednačina

# Površinski napon i temperatura



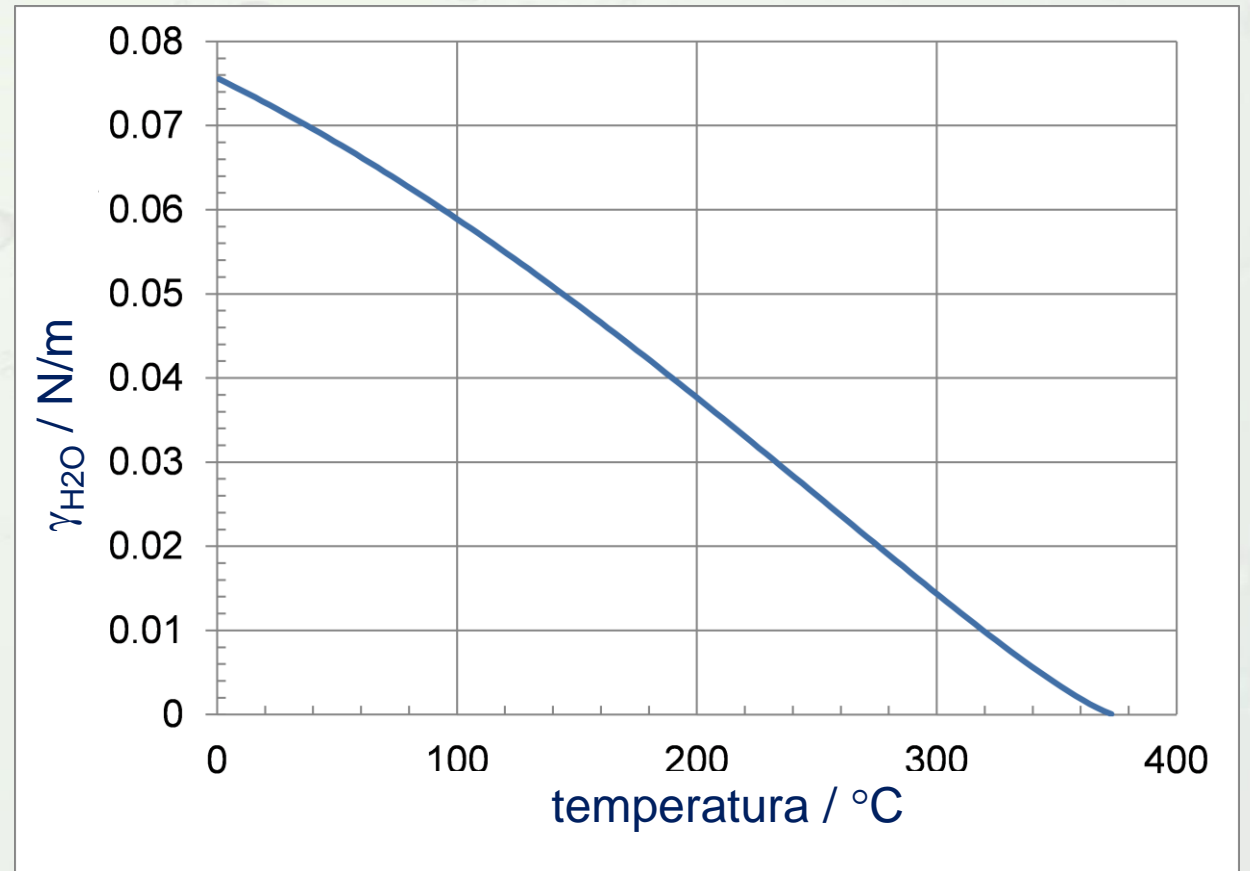
viša temperatura



slabije međumolekulske interakcije



manji  $\gamma$



# Površinski napon odabranih tečnosti (N/m)

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t (°C)	H <sub>2</sub> O	CCl <sub>4</sub>	C <sub>6</sub> H <sub>6</sub>	C <sub>6</sub> H <sub>5</sub> NO <sub>2</sub>	C <sub>6</sub> H <sub>5</sub> OH
0	0,07564	0,0290	0,0316	0,0464	0,0240
25	0,07197	0,0261	0,0282	0,0432	0,0218
50	0,06791	0,0231	0,0250	0,0402	0,0198
70	0,06350	0,0202	0,0219	0,0373	-

# Površinski napon i temperatura

Etveš: 
$$-\frac{d[\gamma V_m^{2/3}]}{dT} = k$$

$$-\int_{\gamma_1 V_{m1}^{2/3}}^{\gamma_2 V_{m2}^{2/3}} d[\gamma V_m^{2/3}] = k \int_{T_1}^{T_2} dT$$

$$-\frac{\gamma_2 V_{m2}^{2/3} - \gamma_1 V_{m1}^{2/3}}{T_2 - T_1} = k$$

$$T_2 = T_c; \quad \gamma_2 = 0$$

$$\boxed{\gamma V_m^{2/3} = k(T_c - T)}$$

Druge empirijske jednačine:

Remzi i Šilds: 
$$\gamma V_m^{2/3} = k(T_c - T - 6)$$

Van der Vals: 
$$\gamma = \gamma_0 \left(1 - \frac{T}{T_c}\right)^n$$

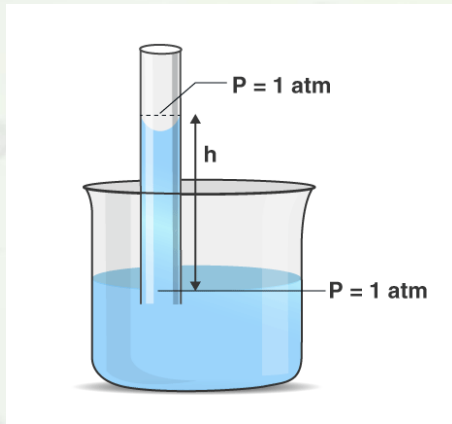
Katajama: 
$$\gamma \left(\frac{M}{\rho - \rho'}\right)^{2/3} = k(T_c - T)$$

Meklod: 
$$\gamma = C(\rho - \rho')^4$$

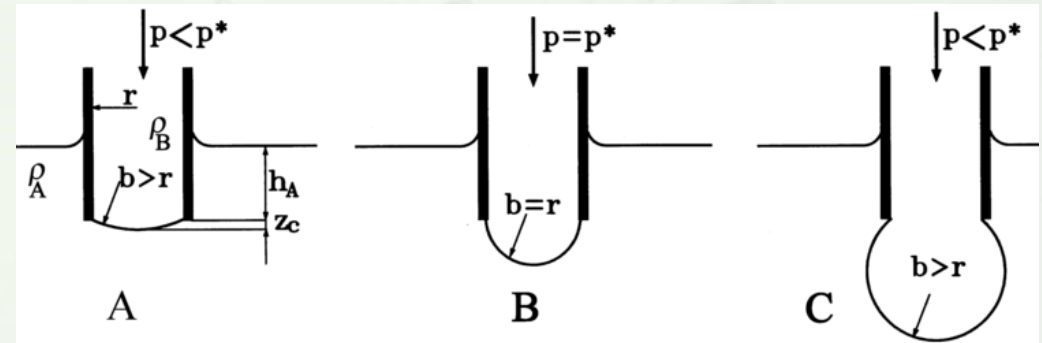


# Merenje koeficijenta površinskog napona

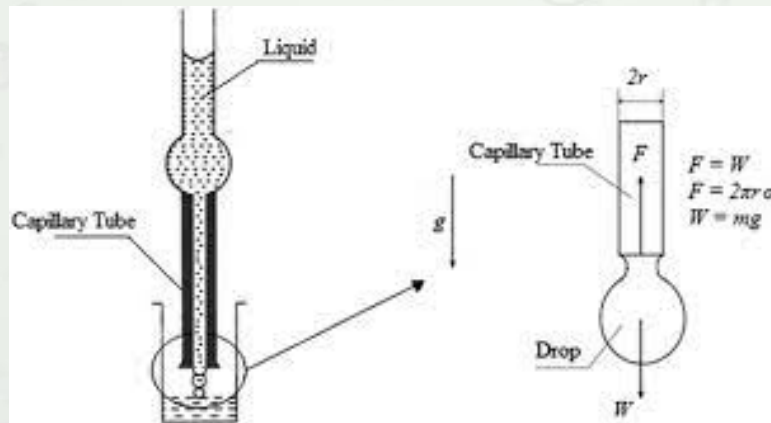
podizanje nivoa tečnosti u kapilari



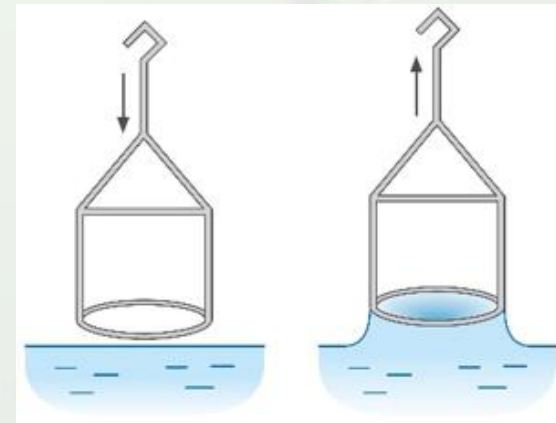
metoda mehura maksimalnog pritiska



stalagmometrijska metoda



tenziometar



# Rezime

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